RADIATIVE HEAT TRANSFER TO A LAMINAR LIQUID FLOW

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A calculation is presented of the temperature distribution in a laminar liquid flow moving in a circular tube. The heat exchange on the outer surface of the channel is determined by the Stefan–Boltzmann law.

Let us investigate the transfer of heat inside a liquid flowing in a circular pipe whose wall has zero resistance. The local heat-flux density on the channel wall is proportional to the difference between the fourth powers of the temperatures of the outside surface and the gaseous medium. Assuming the liquid flow to be stabilized with a parabolic velocity profile, we consider the two cases of greatest practical interest:

1) radiative heating of a gaseous medium with temperature T_m;

2) radiative cooling in a medium of zero temperature.

The mathematical formulation of the first problem includes the energy equation

$$\frac{\partial^2\theta(R,\ X)}{\partial R^2}+\frac{1}{R}\cdot\frac{\partial\theta(R,\ X)}{\partial R}=(1-R^2)\ \frac{\partial\theta(R,\ X)}{\partial X} \tag{1}$$

and the boundary conditions

$$\theta(R,\ 0)=\theta_0, \tag{2}$$

$$\frac{\partial\theta(1,\ X)}{\partial R}=\mathrm{Ki}_1\left[1-\theta^4(1,\ X)\right], \tag{3}$$

$$\frac{\partial\theta(0,\ X)}{\partial R}=0, \tag{4}$$

where

$$R=\frac{r}{r_0};\quad X=\frac{2x}{\mathrm{Pe}\,d_0};\quad \mathrm{Pe}=\frac{\overline{W}d_0}{a};\quad d_0=2r_0;$$

$$\theta=\frac{T}{T_m},\quad \mathrm{Ki}_1=\frac{\sigma_b T_m^3 r_0}{\lambda}\cdot\frac{d}{d_0}.$$

A general analytic method of solving heat-conduction problems with different nonlinear boundary conditions was developed in [1], and was subsequently used to calculate nonstationary radiant heating and cooling of solids [2, 3]. With suitable modifications, this method can be easily generalized to include cases of heat exchange with a flowing liquid.

Thus, the application of the transformation

$$\frac{\ln\vartheta}{-p}=\int\limits_0^\theta\frac{d\theta}{1-\theta^4}=\frac{1}{2}\ (\mathrm{Arth}\,\theta+\mathrm{arctg}\,\theta) \tag{5}$$

to the problem (1)-(4) yields

$$\frac{\partial^2\vartheta}{\partial R^2}+\frac{1}{R}\ \frac{\partial\vartheta}{\partial R}=(1-R^2)\ \frac{\partial\vartheta}{\partial X}+p\vartheta\left(\frac{\partial\theta/\partial R}{1-\theta^4}\right)^2(p-4\theta^3), \tag{6}$$

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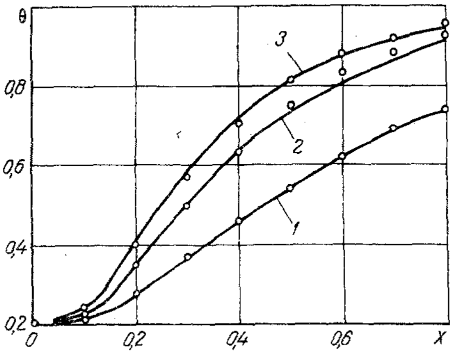

Fig. 1. Variation of the temperature $\theta(0, X)$ in a liquid stream (the curves represent the computer data and the points calculations by the proposed procedure): 1-3) $\mathrm{Ki}_1 = 0.5$, 1.0, and 1.5, respectively.

$$\vartheta(R, 0) = \exp\left[-\frac{p}{2}(\operatorname{Arth}\theta_0 + \operatorname{arctg}\theta_0)\right] = \vartheta_0, \tag{7}$$

$$\frac{\partial\vartheta(1, X)}{\partial R} = -p\,\mathrm{Ki}_1\,\vartheta(1, X), \tag{8}$$

$$\frac{\partial\vartheta(0, X)}{\partial R} = 0. \tag{9}$$

To solve the system (6)-(9) it is necessary first to maintain the nonlinear function $f_1(p, \vartheta, \theta, \partial\theta/\partial R)$ in the right-hand side of the transformed energy equation (6). The condition $f_1 \to 0$ can be realized in the following manner, as indicated in [1, 2]: the region of variation $\theta(R, X)$ is broken up into several segments $(\theta_0 - \theta_1, \ldots, \theta_{i-1} - \theta_i, \ldots)$, in each of which it is assumed that $p_i = 4\theta_i^3$. It should be noted that in the case of moderate heating (not too large values of the Ki_1 number), minimization is ensured by a sufficiently simple choice of the parameter p without subdividing the entire range of temperature variation into intervals:

$$p = 4\left(\frac{\theta_0 + 1}{2}\right)^3. \tag{10}$$

The solution of the linearized equation (6) under the boundary conditions (7)-(9) is

$$\vartheta = \vartheta_0 \sum_{n=0}^{\infty} A_n \psi_n(R) \exp\left(-2\varepsilon_n^2 \frac{1}{\mathrm{Pe}} \cdot \frac{x}{d_0}\right), \tag{11}$$

where A_n, ψ_n, and ε_n are defined in [4].

Substitution of (11) into the transformation (5), which is tabulated in [2], yields the final solution of this problem.

The process of radiative cooling in a medium of zero temperature is described by the energy equation (1) with boundary conditions

$$\theta(R, 0) = 1, \quad \frac{\partial\theta(1, X)}{\partial R} = -\mathrm{Ki}_2\,\theta^4(1, X), \quad \frac{\partial\theta(0, X)}{\partial R} = 0.$$

Here

$$\theta = \frac{T}{T_0}, \quad \mathrm{Ki}_2 = \frac{\sigma_b T_0^3 r_0}{\lambda} \cdot \frac{d}{d_0}.$$

The transformation

$$\vartheta(R, X) = \exp\left[-\frac{p}{3}\theta^{-3}(R, X)\right], \tag{12}$$

without changing the symmetry conditions, linearizes the boundary condition

$$\frac{\partial\vartheta(1, X)}{\partial R} = -p\,\mathrm{Ki}_2\,\vartheta(1, X).$$

This gives rise to the following nonlinear function in the transformed equation for the energy:

$$f_2 = \frac{\partial\vartheta}{\partial R} \cdot \frac{\partial\theta}{\partial R}\,\theta^{-4}(p - 4\theta^3) \tag{13}$$

and

$$\vartheta(R, 0) = \exp\left(-\frac{p}{3}\right) = \vartheta_0.$$

The calculation then follows the same sequence as in the first problem. The nonlinear complex (13) is first linearized by assuming $P_i = 4\theta_i^3$ in each interval $1 - \theta_1, \ldots, \theta_{i-1} - \theta_i, \ldots$. This is followed by the use of a solution of the type (11) and the transformation (12).

The temperatures inside a liquid flowing in a plane channel are calculated in similar fashion in the case of rod flow, in problems with axial heat diffusion, when the stream contains internal heat sources distributed homogeneously or inhomogeneously over the cross section, etc.

A major advantage of this method is its relatively high accuracy at a minimum number of intervals. The temperatures of a liquid on the axis of a round tube, calculated by the method described above, are compared in Fig. 1 with computer data. The calculation was performed for the case of rod flow at $Ki_1 = 0.5$, 1.0, and 1.5. The temperature of the liquid entering the channel was assumed to be $\theta_0 = 0.2$. The abscissa of the diagram is the reduced length of the tube $X = 4/Pe \cdot x/d_0$. The first two heating processes were calculated without subdividing the entire range of temperature variation into intervals, and the correction parameter was determined from formula (10). To find the temperature on the tube axis at $Ki_1 = 1.5$, the range of variation of the sought temperature (from 0.2 to 1.0) was divided into only two intervals, 0.2-0.6 and 0.6-1.0, in which case $p_1 = 4(0.6)^3$ and $p_2 = 4[(0.6 + 1.0)/2]^3$. The difference between our results and the computer data for all three cases was nowhere larger than 3.0-4.0%.

In conclusion we present formulas with which it is possible to calculate the entire temperature field inside a liquid stream if the distribution of the temperatures along the arbitrarily chosen coordinate axes $r = r_*$ and $x = x_*$ is known.

As is well known [4], at a sufficiently large reduced tube length X, all the terms of the series (11) except the first can be neglected, i.e., in general form we have

$$\vartheta(r, x) = Q(r) S(x).$$

Then Eqs. (5) and (12) take the form

$$Q(r) S(x) = \exp - \frac{p}{2} \left[\operatorname{Arth} \theta(r, x) + \operatorname{arctg} \theta(r, x)\right]$$

and

$$Q(r) S(x) = \exp - \frac{p}{3} \theta^{-3}(r, x).$$

If we now trace the temperature variation along the coordinate $r = r_*$, and then $x = x_*$, in analogy with [5, 6] as well as [7], then simple transformations yield

$$\operatorname{Arth} \frac{\theta(r, x) + \theta(r_*, x_*)}{1 + \theta(r, x)\theta(r_*, x_*)} + \operatorname{arctg} \frac{\theta(r, x) + \theta(r_*, x_*)}{1 - \theta(r, x)\theta(r_*, x_*)}$$

$$= \operatorname{Arth} \frac{\theta(r_*, x) + \theta(r, x_*)}{1 + \theta(r_*, x)\theta(r, x_*)} + \operatorname{arctg} \frac{\theta(r_*, x) + \theta(r, x_*)}{1 - \theta(r_*, x)\theta(r, x_*)}, \tag{14}$$

$$\theta^{-3}(r, x) = \theta^{-3}(r_*, x) + \theta^{-3}(r, x_*) - \theta^{-3}(r_*, x_*). \tag{15}$$

The relations (14) and (15) are similar to those obtained in [2, 3] for the nonstationary thermal conductivity. This is due, first, to the fact that the expressions for the temperature field, just as in [2, 3], are limited to the first term of an infinite series, and second to self-similarity of the laminar flow under consideration.

It is important to note that Eqs. (14) and (15), which make it possible to determine the temperature $\theta(r, x)$ at any point of the stream from the measured temperatures and the three points with coordinates r_*, x_*; r_*, x; and r, x_*, do not require knowledge of the thermophysical properties of the liquid, the stream velocity, or the degree of blackness of the exterior surface. In addition, since the origin $0(r_*, x_*)$ can be chosen arbitrarily, it is most convenient to use (14) and (15) when the temperature distribution in the stream must be determined from the values of the known temperatures on the channel wall, i.e., when $r_* = r_0$.

NOTATION

θ	is the relative dimensionless temperature;
R and X	are the generalized coordinates;
Pe	is the Péclet number;
Ki	is the Kirpichev number;
d_0	is the inside diameter of the tube;

d	is the outside diameter of the tube;
$\overline{W}$	is the velocity averaged over the cross section;
λ and a	are the thermal conductivity and thermal diffusivity of liquid;
σ_b	is the radiation coefficient;
p	is the correction parameter.

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